Thomson scattering of coherent diffraction radiation by an electron bunch

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This paper considers the process of Thomson scattering of coherent diffraction radiation (CDR) produced by the preceding electron bunch in the accelerator onto one of the subsequent bunches. It is shown that the yield of scattered hard photons is proportional to N_e^3, where N_e is the number of electrons per bunch. A geometry is chosen for the CDR generation and an expression is obtained for the scattered-photon spectrum, with regard to the geometry used, that depends explicitly on the bunch size. A technique is proposed for measuring the bunch length using scattered radiation characteristics. [S1063-651X(99)03408-X]

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I. INTRODUCTION

The process of Compton backscattering (CBS) of the infrared or visible photons by the relativistic electrons has been used widely for obtaining x-ray and γ beams when the energy ranges from $\sim 10^4$ eV up to $\sim 10^{10}$ eV [1–4].

The development of laser technologies in recent years has raised the prospect of using the CBS process for electron bunch diagnostics [5–7]. The authors of a particular experiment [7] used a femtosecond near-infrared terawatt laser as a source of radiation, which was scattered onto a bunch of electrons with energy $E=50$ MeV. They proposed using this process to measure electron bunch characteristics (longitudinal and transverse bunch sizes, divergence, etc.). The longitudinal bunch structure, for instance, was measured via the dependence of the scattered hard photon yield on the time delay between the electron and photon bunches. It is clear that the accuracy of such measurements relies on the reproducibility and controllability of the characteristics of a powerful laser, which is a rather complicated problem.

In further works [8,9], it was proposed to measure the bunch length through such characteristics of coherent transition radiation (i.e., the transition radiation with a wavelength comparable to the bunch length) as the radiation spectrum and the autocorrelation function. In the latter cases, one is free from the errors associated with the laser. However, the methods so far proposed are not nondestructive (viz. the electron beam crosses the foil target).

This paper considers the possibility of electron beam diagnostics using Thomson scattering of coherent diffraction radiation (CDR) from the preceding bunch onto the subsequent one. Diffraction radiation is produced when a charged particle moves close to a conducting target. The effects of the target on beam characteristics could be reduced to an acceptable level by the choice of geometry (see Conclusion). Thus, the method proposed here is nondestructive, as are the methods involving the use of laser emission; nonetheless, characteristics of the scattered hard radiation are determined only by the electron beam parameters.

II. THOMSON SCATTERING OF RADIATION BY A MOVING BUNCH

During the interaction of an incident photon with a moving electron, the scattered-photon energy is derived using the conservation laws:

$$\omega_2=\omega_1\frac{1-\beta\cos\theta_1}{1-\beta\cos\theta_2+\frac{\omega_1}{E}\{1-\cos(\theta_1-\theta_2)\}}. \qquad (1)$$

Here ω_1, ω_2, and E are the energies of the incident and scattered photons and that of the electron, respectively; $\beta=v/c$; v is the electron velocity; the angles between the electron momentum and the incident and scattered photons, θ_1,θ_2, are the same as in [6]. If the primary-photon energy and that of the electron satisfy the conditions

$$\gamma=E/mc^2\gg 1, \quad \gamma\omega_1\ll mc^2, \qquad (2)$$

the scattering-photon energy (1) is linearly dependent on that of the incident photon:

$$\omega_2=\omega_1\frac{1-\beta\cos\theta_1}{1-\beta\cos\theta_2}\approx\omega_1\frac{2\gamma^2(1-\beta\cos\theta_1)}{1+(\gamma\theta_2)^2}, \qquad (3)$$

where the outgoing photon angle $\theta_2\sim\gamma^{-1}$.

In a frame where the electron is at rest [electron rest frame (ERF)], the energy of the photon scattered is, according to Eq. (2), significantly less than the electron mass. The photon scattering then occurs virtually without any frequency change and, therefore, the scattering process may be described in terms of classical electrodynamics (Thomson scattering).

In the ERF, the classical cross section of scattering of an electromagnetic wave by a free charge [10] is not controlled by its frequency and is given by the expression

$$\frac{d\sigma}{d\Omega'}=\frac{r_0^2}{2}(1+\cos^2\theta'). \qquad (4)$$

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In Eq. (5), $r_o = 2.82\times 10^{-13}$ cm is the classical radius of an electron, and the primes denote the angles in the ERF. Transforming these to the laboratory system, we have

$$\cos\theta' = \frac{\cos\theta_2 - \beta}{1-\beta\cos\theta_2}, \tag{5}$$

$$d\Omega' = \frac{1-\beta^2}{(1-\beta\cos\theta_2)^2} d\Omega. \tag{6}$$

From Eqs. (5) and (6) we obtain the classical cross section for the ultrarelativistic case:

$$\frac{d\sigma}{d\Omega} = 4r_0^2\gamma^2 \frac{1+(\gamma\theta_2)^4}{[1+(\gamma\theta_2)^2]^4}. \tag{7}$$

The total cross section derived through integrating expression (7) with respect to angles is the Thomson cross section:

$$\sigma_T = \frac{8}{3}\pi r_0^2. \tag{8}$$

The yield of secondary photons upon scattering, e.g., of incident laser photons, onto a moving electron bunch is to be determined not only by the cross section of the process but also by the overlapping of the laser and electron beams in space and time, which is characterized by luminosity L:

$$\frac{dN_2}{dt} = L\sigma_T. \tag{9}$$

Let us consider the head-on collision of electron and photon bunches. Luminosity in this case is defined as follows:

$$L = cN_eN_{ph}F \int\int\int\int dx\,dy\,dz\,dt\, f_{ph}(x,y,z+ct) \times f_e(x,y,z-\beta ct). \tag{10}$$

Here N_e and N_{ph} are the number of particles in the electron and photon bunches, f_e and f_{ph} are the corresponding normalized electron and photon distributions, and F is the collision frequency of the bunches. For the monodirected beams with a Gaussian distribution in both transversal and longitudinal directions,

$$f_e = \frac{2}{(2\pi)^{3/2}\ \sigma_e^2 l_e} \exp\left\{ -\frac{r^2}{\sigma_e^2} - \frac{(z-\beta\cot)^2}{2l_e^2} \right\},$$

$$f_{ph} = \frac{2}{(2\pi)^{3/2} \sigma_{ph}^2 l_{ph}} \exp\left\{ -\frac{r^2}{\sigma_{ph}^2} - \frac{(z+\cot)^2}{2l_{ph}^2} \right\}, \tag{11}$$

$$r^2 = x^2 + y^2,$$

the luminosity is readily calculated:

$$L = N_eN_{ph}F \frac{1}{2\pi(\sigma_e^2+\sigma_{ph}^2)}. \tag{12}$$

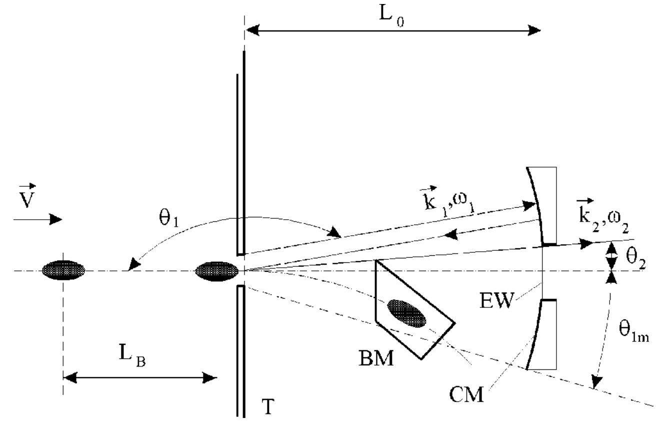

FIG. 1. Scheme for Thomson scattering of CDR from a circular aperture. T, conducting target; BM, bending magnet; CM, concave mirror; EW, exit window.

In Eq. (11), σ_e^2 and σ_{ph}^2 are the variables characterizing the transversal and l_e^2 and l_{ph}^2 are those for the longitudinal distributions. For the head-on collisions, it follows from Eq. (12) that the luminosity is governed solely by the transverse dimensions of the electron and photon bunches. The number of photons scattered through the collision of single bunches can be estimated from Eqs. (9) and (12):

$$N_2 = \frac{1}{2} N_{ph} \frac{N_e\sigma_T}{S_e + S_{ph}}, \tag{13}$$

where S_e and S_{ph} are the cross-sections of the electron and photon bunches. The value $S = \frac{1}{2}N_e[\sigma_T/(S_e+S_{ph})]$ can be treated as the reflectivity of the electron bunch. For the electron numbers and bunch size attainable, this value is considerably small. Therefore, one typically uses radiation of a powerful laser as a primary beam.

However, effective overlapping of the laser and accelerator bunches is a difficult task, while linear dependence of the scattered-beam intensity (8) on the number of electrons in the bunch poses natural restrictions on the intensity of the resulting x-ray or γ beam. If a beam of incident photons is to be generated by one of the preceding electron bunches in the accelerator, then the temporal and longitudinal structures of the colliding bunches will be similar.

It seems possible that one can use a beam of coherent radiation of a short electron bunch as a primary beam of soft photons. In this case, the radiation intensity in the wavelength region λ_1, comparable to the bunch length, is quadratically dependent on the number of electrons in the bunch [11], which compensates for the low reflectivity of the electron bunch. Instead of a laser source, CDR, i.e., the radiation produced while a short bunch of electrons is passing close to a metal target [12], can be taken as a source of primary radiation.

Figure 1 shows a potential experiment scheme. Electron bunches pass through a circular opening of radius R in a metal target, which results in the generation of CDR in the wavelength region $\lambda_1 \geqslant l_e$, the electrons are deflected by a bending magnet (BM), while CDR is reflected and focused by a thin concave mirror (CM) on one of the following bunches. The scattered photons with energy corresponding to the x-ray region are extracted through the center hole of the

CM, suffering only a small loss. The distance between the center hole of the mirror and the target, L_o, is selected from the condition

$$2L_o=\frac{L_B}{\beta}m, \quad m=1,2,3\ \ldots, \tag{14}$$

where L_B is the distance between bunches.

The spectrum of the photons backscattered by a single electron may be calculated in the following manner:

$$\frac{dN_2^0}{d\omega_2}=\text{const}\int\int d\Omega_2 d\omega_1 \frac{dN_1}{d\omega_1}\frac{d\sigma}{d\Omega_2} \times\delta\left(\omega_2-\omega_1\frac{4\gamma^2}{1+(\gamma\theta_2)^2}\right). \tag{15}$$

Here $dN_1/d\omega_1$ is the spectrum of the incident photon beam. Integration in Eq. (15) should be carried out with respect to all the spectral region of the initial radiation and the exit aperture $\Delta\Omega_2$.

The yield of photons scattered by an electron bunch is described by a more complicated formula:

$$\frac{dN_2^B}{d\omega_2}=\int\int d\Omega_2 d\omega_1 \frac{dN_1}{d\omega_1}\frac{d\sigma}{d\Omega_2}\frac{N_e}{2\pi(\sigma_e^2+\sigma_{ph}^2)} \times\delta\left(\omega_2-\omega_1\frac{4\gamma^2}{1+(\gamma\theta_2)^2}\right). \tag{16}$$

III. SPECTRUM OF COHERENT DIFFRACTION RADIATION

DR spectra may be calculated numerically using the results of previous works [12,13] for the spectral angular density of the energy radiated from a single electron passing through a circular opening with radius R in an ideal conductor:

$$\frac{d^2W}{dx d\Omega}=\frac{\alpha\omega_c}{\pi^2}\frac{\sin^2\theta}{(\sin^2\theta+\gamma^{-2})^2}J_0^2\left(\frac{x}{2}\gamma\sin\theta\right)K_1^2\left(\frac{x}{2}\right)\left(\frac{x}{2}\right), \tag{17}$$

where α is the fine-structure constant, $\omega_c=\gamma/2R$ is the characteristic energy of DR, θ is the outgoing photon angle, ω_1 is the energy of the emitted photon, and $x=\omega_1/\omega_c$ is the dimensionless energy variable. From here on in this paper we will use the system of units $\hbar=m=c=1$.

In expression (17), $J_0(x)$ is the Bessel function of the zeroth order and $K_1(x)$ is the modified Bessel function. From Eq. (17) one may obtain the DR intensity spectrum dW/dx after integration with respect to the solid angle covered by the reflected mirror. Calculated spectra for apex angles $\theta_{1m}=k_1/\gamma$ $(k_1=5,10)$ are shown in Fig. 2.

Following [11], one may write the spectrum of CDR emitted by a bunch of N_e electrons as follows:

$$\frac{dN_1^B}{d\omega_1}=N_e(1+f(\lambda_1)N_e)\frac{dN^0}{d\omega_1}\approx N_e^2 f(\lambda_1)\frac{dN^0}{d\omega_1}, \quad \lambda_1\geqslant l_e. \tag{18}$$

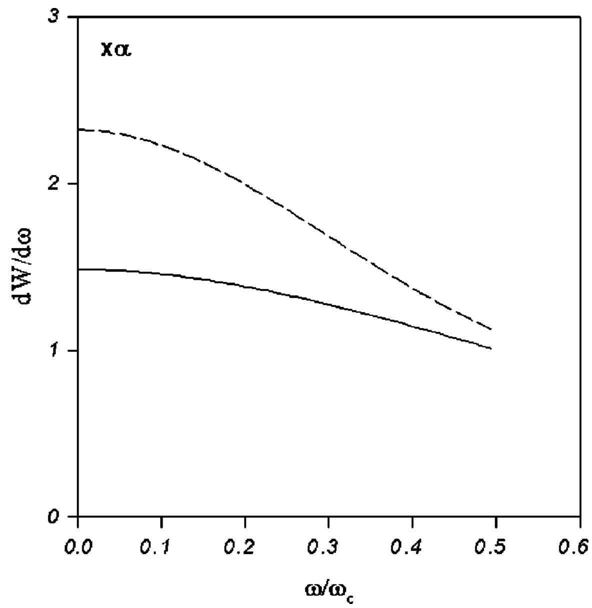

FIG. 2. Diffraction radiation intensity spectrum from a circular aperature (lower curve is for $k_1=5$; upper curve, $k_1=10$).

Here λ_1 is the DR wavelength and $f(\lambda_1)$ is the bunch form factor, defined as the squared Fourier transform of longitudinal distribution of electron density in a bunch. For the Gaussian distribution (11) we have

$$f(\lambda_1)=\left|\frac{1}{\sqrt{2\pi}l_e}\int\exp\left(-\frac{z^2}{2l_e^2}\right)\exp\left(-i\frac{2\pi z}{\lambda_1}\right)dz\right|^2 =\exp\left(-\frac{4\pi^2 l_e^2}{\lambda_1^2}\right)=\exp(-\omega_1^2 l_e^2). \tag{19}$$

The photon DR spectrum may be easily derived from the DR intensity spectrum:

$$\frac{dN^0}{d\omega_1}=\frac{1}{\omega_1}\frac{dW}{d\omega_1}=\frac{1}{\omega_1}\frac{dW}{\omega_c dx}. \tag{20}$$

It is clear that there are two energies characterizing the spectrum (18):

$$\omega_{ch1}=\omega_c=\frac{\gamma}{2R}, \quad \omega_{ch2}=\frac{2\pi}{l_e}; \tag{21}$$

one of them, ω_{ch1}, connected with the DR spectrum from a single electron and the other (ω_{ch2}) with the collective emission from the bunch.

For an ultrarelativistic electron beam, the transversal and longitudinal sizes of a bunch may be less than 1 mm. In a similar case, one may use a hole with the a radius R of approximately a few millimeters. So, we may consider the case

$$\frac{\gamma}{2R}\geqslant\frac{1}{l_e}. \tag{22}$$

This means that the coherent effects are significant in the region $\omega_1 \ll \omega_c$ where the DR intensity spectrum may be taken as a constant (see Fig. 2). In the limit $\omega_1 \to 0$ $(x \to 0)$ we have

$$\frac{dW}{d\omega_1} \approx \frac{\alpha}{\pi}\left\{ \ln(1+k_1^2) - \frac{k_1^2}{1+k_1^2} \right\} = \frac{\alpha}{\pi} C_\parallel . \tag{23}$$

After all substitutions, one may obtain

$$\frac{dN_2^B}{d\omega_2} = \frac{2}{\pi^2}\alpha r_0^2 N_e^3 C_\parallel \int\int d\omega_1 d\Omega_2 \frac{1}{\omega_1} \times \frac{\gamma^2[1+(\gamma\theta_2)^4]}{[1+(\gamma\theta_2)^2]^4} \frac{\exp(-\omega_1^2 l_e^2)}{(\sigma_e^2+\sigma_{ph}^2)} \times \delta\left(\omega_2 - \omega_1 \frac{4\gamma^2}{1+(\gamma\theta_2)^2} \right). \tag{24}$$

In formula (24) the denominator has the value σ_{ph}^2 characterizing the radius of the focused photon beam in the interaction point. Because of the diffraction limit, the size of the light spot cannot be less than $\lambda_1/2\pi$. So, for estimations we shall use the latter value instead of σ_{ph}.

As one can see from Eq. (24), the scattered yield has cubic dependence on the number of electrons per bunch. Other authors [14,15] considered electromagnetic radiation produced by the collision of short electron bunches and also arrived at a cubic dependence of the photon yield with the energy $\omega < 4\gamma^2/l_e$ during collision of identical bunches.

Roughly speaking, the works mentioned earlier studied scattering of the field of virtual photons of one bunch on the other, while this paper deals with the process in which real photons emitted by the preceding bunch are scattered onto one of the subsequent bunches.

IV. DEPENDENCE OF CHARACTERISTICS OF SCATTERED PHOTONS ON ELECTRON BUNCH PARAMETERS

Because the narrow angular distribution of the scattered photons decreases as $(\gamma\theta_2)^{-4}$ for large emission angle $\theta_2 \gg \gamma^{-1}$, Eq. (24) may be simplified to

$$\frac{dN_2^B}{d\omega_2} = \frac{2}{\pi^2}\alpha r_0^2 N_e^3 \gamma^2 \Delta\Omega_2 C_\parallel \int d\omega_1 \delta(\omega_2 - 4\gamma^2\omega_1) \times \frac{\exp(-\omega_1^2 l_e^2)}{\omega_1\left(\sigma_e^2 + \frac{1}{\omega_1^2}\right)} \tag{25}$$

if the exit aperture $\Delta\Omega_2 = \pi\theta_{2\max}^2$ is comparable to γ^{-2}:

$$\theta_{2\max} = k_2\gamma^{-1}, \quad k_2 \sim 1.$$

Using the well known property of the δ function, we may obtain

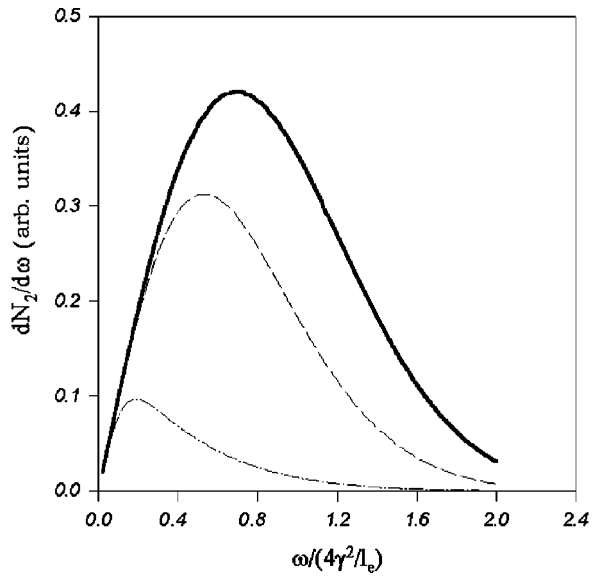

FIG. 3. Spectrum of the scattered photons for the scheme shown in Fig. 1 ($r = \sigma_c/1_c = 5$, dashed-dotted line; $r = 1$, dotted line; $r = 0.2$, solid line).

$$\frac{dN_2}{d\omega_2} = \frac{2}{\pi}\alpha r_0^2 N_e^3 C_\parallel k_2^2 \frac{\exp\left[-\left(\frac{\omega_2 l_e}{4\gamma^2}\right)^2\right]}{\omega_2\left[\sigma_e^2 + \left(\frac{4\gamma^2}{\omega_2}\right)^2\right]} = \frac{2}{\pi}\alpha r_0^2 N_e^3 C_\parallel k_2^2 \frac{F}{\omega_2}. \tag{26}$$

One may see from Eq. (26) that the yield of scattered photons does not depend on the electron energy if the maximum outgoing angle $\theta_{2\max}$ is measured in units γ^{-1}. Of course, the scale of transformation of the photon energy is defined by the electron energy [see Eq. (3)].

The spectrum (26) is shown in Fig. 3 for different ratios between σ_e and l_e. Clear, broad maxima whose positions are determined by the ratio $r = \sigma_e/l_e$ are presented. As this ratio decreases, the spectral maximum shifts to the value

$$\omega_{2max} = \frac{1}{\sqrt{2}} \frac{4\gamma^2}{l_e}$$

and the intensity rises due to increasing luminosity. Let us estimate the photon yield at the maximum for the following parameters: $N_e = 10^{10} e^-$/bunch; $\sigma_e = l_e = 1$ mm; $k_1 = 10$ $(C_\parallel = 3.6)$; $k_2 = 3$; $\Delta\omega_2/\omega_2 = 10\%$.

Then

$$\Delta N_2^B = \frac{dN_2^B}{d\omega_2}\Delta\omega_2 = \frac{2\alpha}{\pi}\left(\frac{r_0}{l_e}\right)^2 N_e^3 C_\parallel k_2^2 F_{max} \frac{\Delta\omega_2}{\omega_2} = 3.7 \times 10^4 \ \text{ph/bunch}.$$

For the electron energy E = 1000 MeV the photons scattered at the spectral maximum have an energy of about 1.6 keV.

However, the estimation of the yield obtained above is valid only if the focusing mirror is located at a long distance from the target.

$$L_0 \gg L_f . \tag{27}$$

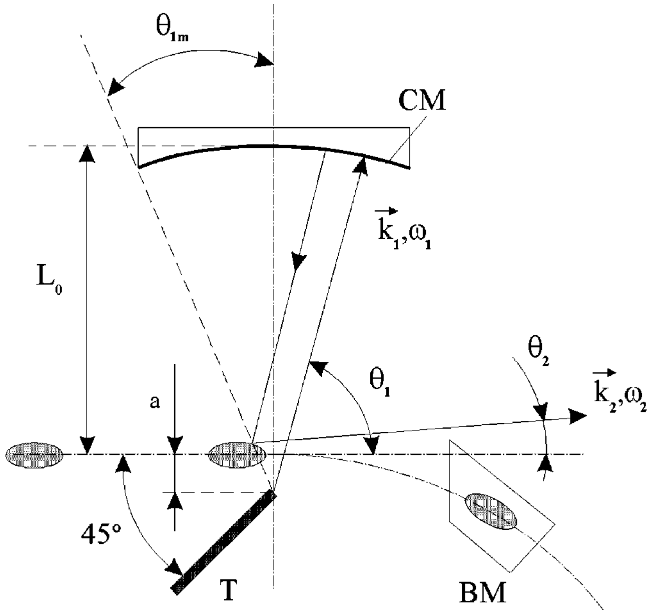

FIG. 4. Scheme for Thomson scattering of CDR from the edge of a tilted target.

Here L_f is the formation length that characterizes the distance at which the radiation of the wavelength λ, propagating at the angle θ, is completely separated from the initiating charge:

$$L_f = \frac{\beta\lambda}{1-\beta\cos\theta}. \tag{28}$$

For forward emission ($\theta_1 \sim \gamma^{-1}$) in the ultrarelativistic case ($\gamma \geqslant 10^2$), the CDR formation length,

$$L_f \approx \frac{2\gamma^2\lambda_1}{1+\gamma^2\theta_1^2}, \tag{29}$$

can exceed tens of meters. In a real case, the mirror CM (Fig. 1) can be placed at a distance $L_0 \ll L_f$. Then the DR intensity (initial photon flux) is suppressed as $(L_f/L_0)^2$ [16]. For the case considered, the suppression factor may reach $\sim 10^{-4}$–10^{-5} for a distance of about a few meters between target and mirror.

But for $\gamma \sim 10^2$ the suppressed factor will be about $\sim 10^{-2}$ only with maximum positioned at $\omega_2 \sim 10$ eV. It means, in principle, that the yield of scattered photons in the visible and UV regions ($\sim 10^2$ photons/bunch) may be detected and used for diagnostics.

As follows from Eq. (28), for the emission angles $\theta_1 \sim \pi/2$ the formation length is comparable to the wavelength. For these large emission angles, the mirror positioned at $L_0 \gg \lambda_1$ does not effect the DR intensity. Figure 4 shows the schematic of a potential application of the proposed geometry. An electron beam passes through the vicinity of a metal target tilted at $\theta = 45°$ with respect to the electron momentum, and CDR propagates at $\theta_1 \approx 90°$ to the beam (in close analogy with backward transition radiation [17]).

Spectral-angular distribution of DR when a single charge passes near a tilted ideally conducting semiplane was obtained in [18]. For the ultrarelativistic case, when we introduce the angles θ_x and θ_y measured from the direction of mirror reflection (the x axis is oriented along the target edge), the spectral-angular distribution of DR is written in a simpler form [19]:

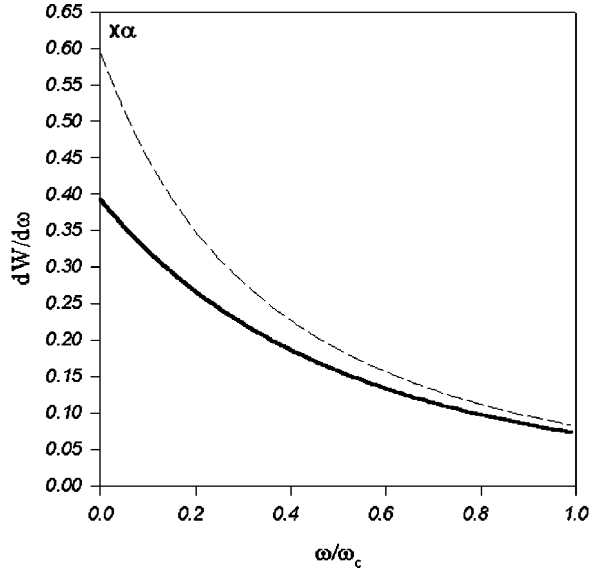

FIG. 5. Diffraction radiation intensity spectrum from the edge of a tilted target (lower curve, $k_1 = 5$; upper curve, $k_1 = 10$).

$$\frac{d^2W}{d\omega_1 d\Omega} = \frac{\alpha}{4\pi^2}\exp\left(-\frac{\omega_1}{\omega_c}\sqrt{1+\gamma^2\theta_x^2}\right) \times \frac{\gamma^{-2}+2\theta_x^2}{(\gamma^{-2}+\theta_x^2)(\gamma^{-2}+\theta_x^2+\theta_y^2)}. \tag{30}$$

Here $\omega_c = \gamma/2a$, where a is the spacing between the particle trajectory and the edge of the target.

Figure 5 shows the DR intensity spectrum, $dW/d\omega_1$, obtained by integrating expression (30) with respect to the focusing mirror aperture $\theta^2 = \theta_x^2 + \theta_y^2 \leqslant (k_1\gamma^{-1})^2$ for $k_1 = 5$ and 10. In contradiction to the case where the beam passes through the center of the hole, the spectrum $dW/d\omega_1$ in the energy range $\omega_1 \ll \omega_c$ will be aproximated by a linear dependence:

$$\frac{dW_1}{d\omega_1} = \frac{\alpha}{\pi}C_\perp\left(1-B(\theta_{1m})\frac{\omega_1}{\omega_c}\right),$$

$$C_\perp = \frac{\alpha}{2\pi}\left\{\ln(1+k_1^2)+\frac{1}{\sqrt{1+k_1^2}}-1\right\}, \tag{31}$$

where $B(5\gamma^{-1}) \approx 2.6$.

The luminosity for the 90° collision of bunches described by distributions (11) can also be calculated analytically:

$$L = cN_eN_{ph}F\int\int\int\int dx\,dy\,dz\,dt\,f_{ph}(x,y,z+ct)f_e \times(x,y+\beta ct,z)$$

$$= \frac{N_eN_{ph}F}{\pi\sqrt{(\sigma_e^2+\sigma_{ph}^2)(\sigma_e^2+\sigma_{ph}^2+2l_{ph}^2+2l_e^2)}}. \tag{32}$$

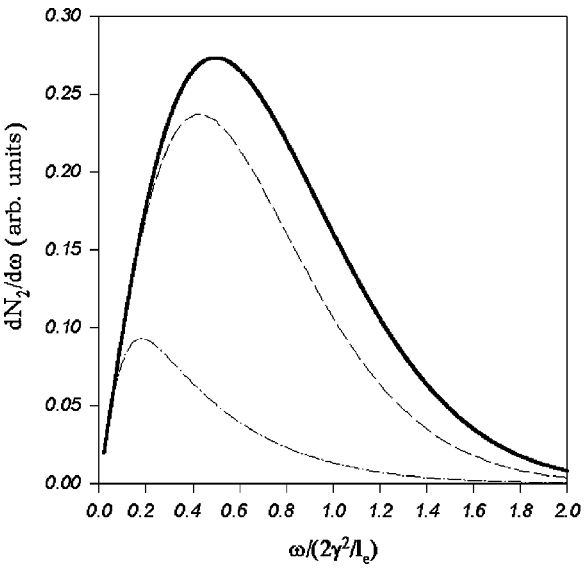

FIG. 6. Spectrum of the scattered photons for the scheme shown in Fig. 4 ($r=5$, dashed-dotted line; $r=1$, dotted line; $r=0.2$, solid line).

Using the same approximations as were used to derive expression (26), we can arrive at

$$\frac{dN_2^B}{d\omega_2}=\frac{4}{\pi}\alpha r_0^2 N_e^3 C_\perp k_2^2 \times\frac{\exp\left[-\left(\frac{\omega_2 l_e}{2\gamma^2}\right)^2\right]}{\omega_2\sqrt{\left[\sigma_e^2+\left(\frac{2\gamma^2}{\omega_2}\right)^2\right]\left[\sigma_e^2+\left(\frac{2\gamma^2}{\omega_2}\right)^2+4l_e^2\right]}}$$
$$=\frac{4}{\pi}\alpha\left(\frac{r_0}{l_e}\right)^2 N_e^3 C_\perp k_2^2 \times\frac{\exp\left[-\left(\frac{\omega_2 l_e}{2\gamma^2}\right)^2\right]}{\omega_2\sqrt{\left[r^2+\left(\frac{2\gamma^2}{l_e\omega_2}\right)^2\right]\left[r^2+4+\left(\frac{2\gamma^2}{l_e\omega_2}\right)^2\right]}}. \quad (33)$$

For the geometry considered, the coefficient of frequency transformation is half that of the head-on collision [see formula (3)].

Depicted in Fig. 6 is the scattered-photon spectrum calculated following formula (33). Similarly to the head-on collision, the spectrum has a maximum in the region of energies

$$\omega_{2m}\approx 0.5\frac{2\gamma^2}{l_e}.$$

Estimation of the scattering-photon yield for the geometry considered here for the same conditions as before gives a close value:

$$\Delta N_2^B=2.9\times 10^4 \ \ \text{photons/bunch}.$$

In contrast to the geometry used previously, however, in this case the radiation-forming length coincides with the wavelength ($\lambda_1\sim 1$ mm). Therefore, having the focusing mirror positioned at a distance $L_0\gg\lambda_1$ would not cause any suppression of the DR yield, and the resulting expression (33) could be used for estimation of the hard photon yield when planning an experiment.

Noteworthy is the fact that when calculating the luminosity (32), it was assumed that the centers of the photon and electron bunches pass the interaction point at the same time. Should the focusing mirror be placed with a certain error ΔL_0, there would appear an additional term in expressions (32) and (33):

$$D(\Delta L_0)=\exp\left(-\frac{\Delta L_0^2}{\sigma_e^2+\sigma_{ph}^2+2(l_e^2+l_{ph}^2)}\right). \quad (34)$$

For the frequent case $\sigma_e<l_e$, one can get the information on the electron bunch length by measuring the scattered-photon yield versus ΔL_0 (detuning curve), since $l_{ph}=l_e$.

V. CONCLUSION

As discussed above, the energy of scattered photons for the case of ultrarelativistic electrons ($\gamma\geqslant 1000$) corresponds to the x-ray region, while for moderate relativistic energies ($\gamma\leqslant 100$) the secondary photon spectrum would include the visible range. It is known that the common techniques for electron-beam diagnostics based on detection of optical transition radiation do not allow us to measure the length of submillimeter bunches. In this context, measurement of the detuning curve by mechanical displacement of the focusing mirror seems to offer a means by which to measure even shorter bunches with the use of simpler equipment than a streak camera.

Rough estimation of the target effect on the beam divergence can be made in the following manner. Let us consider the geometry for $\theta_1=90°$ (Fig. 4). In this case, the DR spectrum for a single electron calculated for the whole radiation cone (around the specular direction) is shown in Fig. 7.

After approximation of the spectrum by the two exponents

$$\frac{dW_{DR}}{d\omega}=\begin{cases}1.28\alpha\exp\left(-5.88\frac{\omega}{\omega_c}\right), & \omega\leqslant 0.2\omega_c\\ 0.48\alpha\exp\left(-1.53\frac{\omega}{\omega_c}\right), & \omega>0.2\omega_c,\end{cases} \quad (35)$$

the radiation losses may be easily calculated:

$$W_{DR}=\int\frac{dW_{DR}}{d\omega}d\omega=0.382\alpha\omega_c. \quad (36)$$

Comparing the value obtained with the exact result [18,19], $W_{DR}=\frac{3}{8}\alpha\omega_c$, one may deduce that the exponential approximation is quite good.

The CDR spectrum per electron moving inside the bunch with the total number of electrons N_e and length l_e instead of Eq. (35) is expressed as

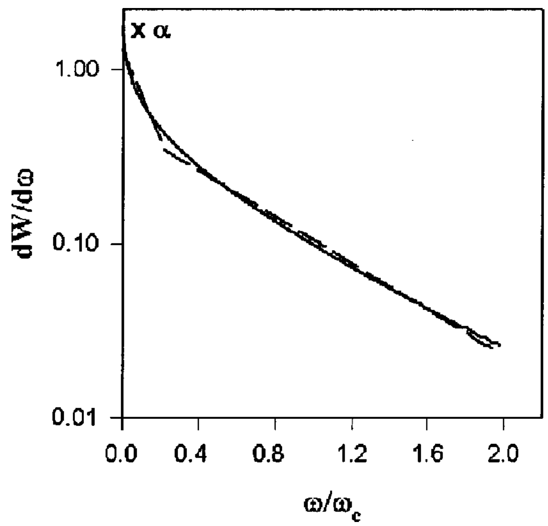

FIG. 7. DR spectrum into one cone from a single electron. Solid line, calculation using Eq. (22); dotted line, approximation (35).

$$\frac{dW_{CDR}}{d\omega}=\begin{cases} N_e\dfrac{dW_{DR}}{d\omega}, & \omega\leqslant\dfrac{2\pi}{l_e} \\ \dfrac{dW_{DR}}{d\omega}, & \omega>\dfrac{2\pi}{l_e}. \end{cases} \tag{37}$$

The energy lost by each electron due to emission along the direction $\theta_1=90°$ is the following:

$$W_{CDR}=\int\frac{dW_{CDR}}{d\omega}d\omega\approx\frac{2.56\pi}{l_e}N_e\alpha, \tag{38}$$

if $N_e\gg 1$, $2\pi/l_e\ll\omega_c$.

Assuming that the total momentum of the CDR photons emitted by an electron is equal to the transversal electron momentum $p_\perp$ that appears after passing near the target, the estimation of the electron deviation angle may be obtained:

$$\theta_e\sim\frac{p_\perp}{p}=\frac{2.56\pi}{l_e}\frac{N_e\alpha}{\gamma}\sim\frac{N_e r_0}{\gamma l_e}=0.03 \text{ mrad} \tag{39}$$

for the beam parameters used before.

The estimation (39) is close to the expression for the so-called geometric wake effect for a beam passing through a collimator [20]. For the case considered $(2\pi/l_e\ll\gamma/2a)$, there is no dependence of the deviation angle on the impact parameter a and one may choose the latter in the range

$$\sigma_e\ll a\ll\frac{\gamma l_e}{4\pi} \tag{40}$$

(for instance, $a=10$ mm if $\gamma=1000$, $l_e=\sigma_e=1$ mm).

The estimation (39) is valid for $\theta_1=90°$. With decreasing θ_1, the component $p_\perp$ equal to the transversal projection of the total CDR photon momentum will diminish as $\sin\theta_1$. Indeed, this consideration is valid for $\theta_1\gg\gamma^{-1}$.

Therefore, the beam divergence growth may be far less than (39) if $\theta_1<30°$.

It should be noted that the CBS process involving laser photons on an electron bunch was considered in 90° geometry [21], and it was shown that for a certain geometry and bunch parameters, the yield of scattered photons may be *much* greater than the yield achieved by scattering on N_e electrons independent of each other. The enhancement factor, dictated by the coherent compton scattering, is proportional to $N_e(\lambda_1/\gamma^2)$. It is to be expected that during scattering of CDR on the subsequent electron bunch, the effect of coherence could be made manifest in a more pronounced fashion, since the wavelength of primary radiation is 2–3 orders of magnitude higher than that of laser emission and, secondarily, since the coherent Thomson scattering would require that the dependence of the number of secondary photons on the number of electrons per bunch be proportional to N_e^4.

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